

## MOPED – Blue skies to brains; astronomical techniques for medical imaging

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STFC Modelling and Simulation Workshop. London. 2 July 2008

with Ben Panter, Rob Tweedie, Mark Bastin, Will Hossack, Trevor Whittley

# Outline

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- ▶ **MOPED is a patented algorithm, which is applicable for the following sorts of problems:**
  - ▶ Problems where rapid, accurate analysis is required
  - ▶ Problems with many more data points than parameters
  - ▶ Problems where parameters need to be estimated from data



# What is it for?

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- ▶ Analysing data very fast, without compromising accuracy
- ▶ Method was developed for analysing galaxy spectra, but is very general
- ▶ **MOPED** stands for *Massively Optimised Parameter Estimation and Data compression*



# Inverse problems

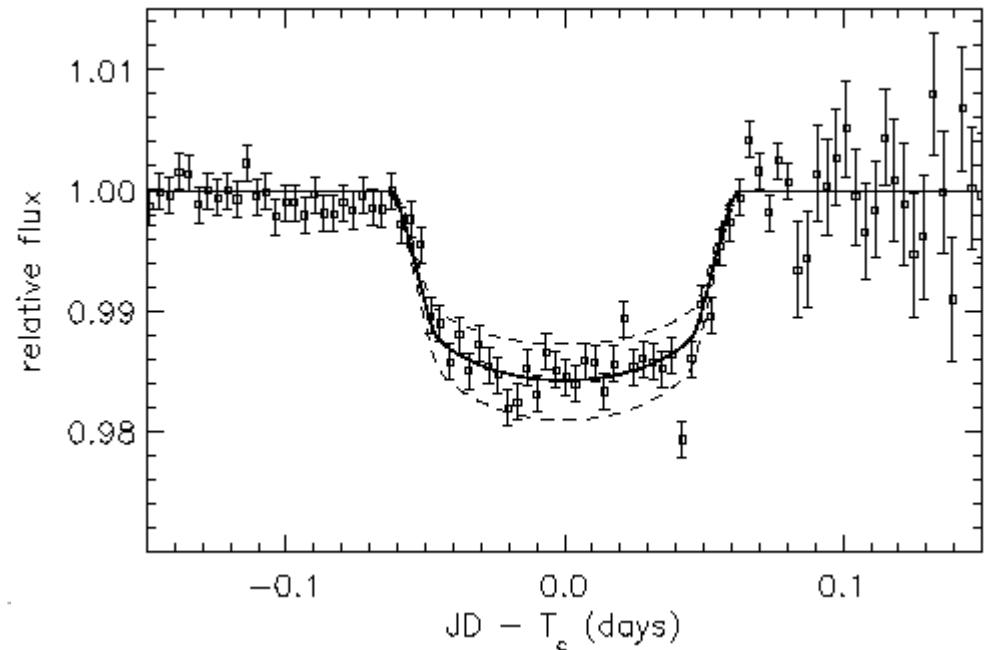
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- ▶ Typically, one has a set of *data*, from which one wants to learn something
- ▶ Very often, the data can be described by a *model*, which contains some *parameters*.
- ▶ The goal is to estimate the parameters, as accurately as possible, and fast



# Example: planet transits

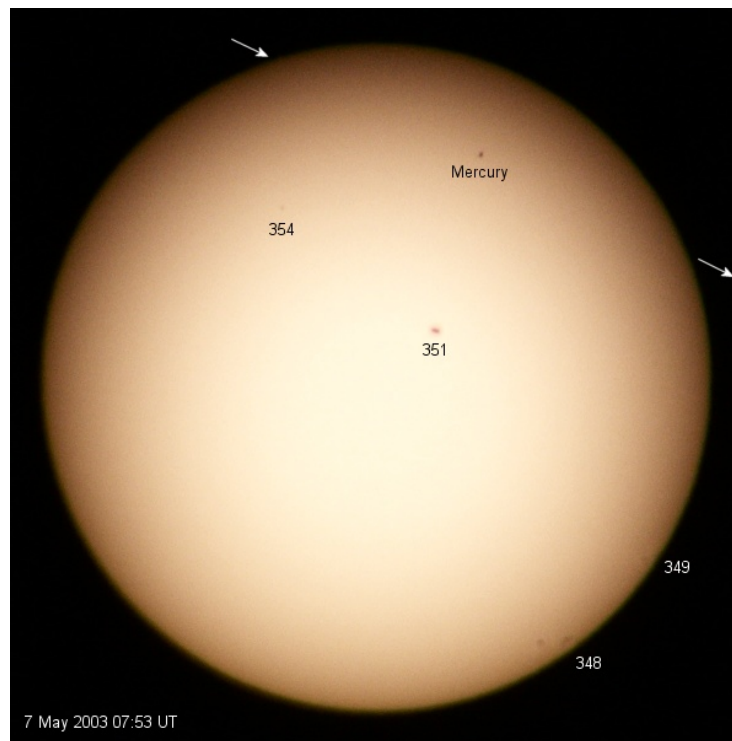
- ▶ **Data:** a light curve = sequence of **N** brightness measurements



# The Model

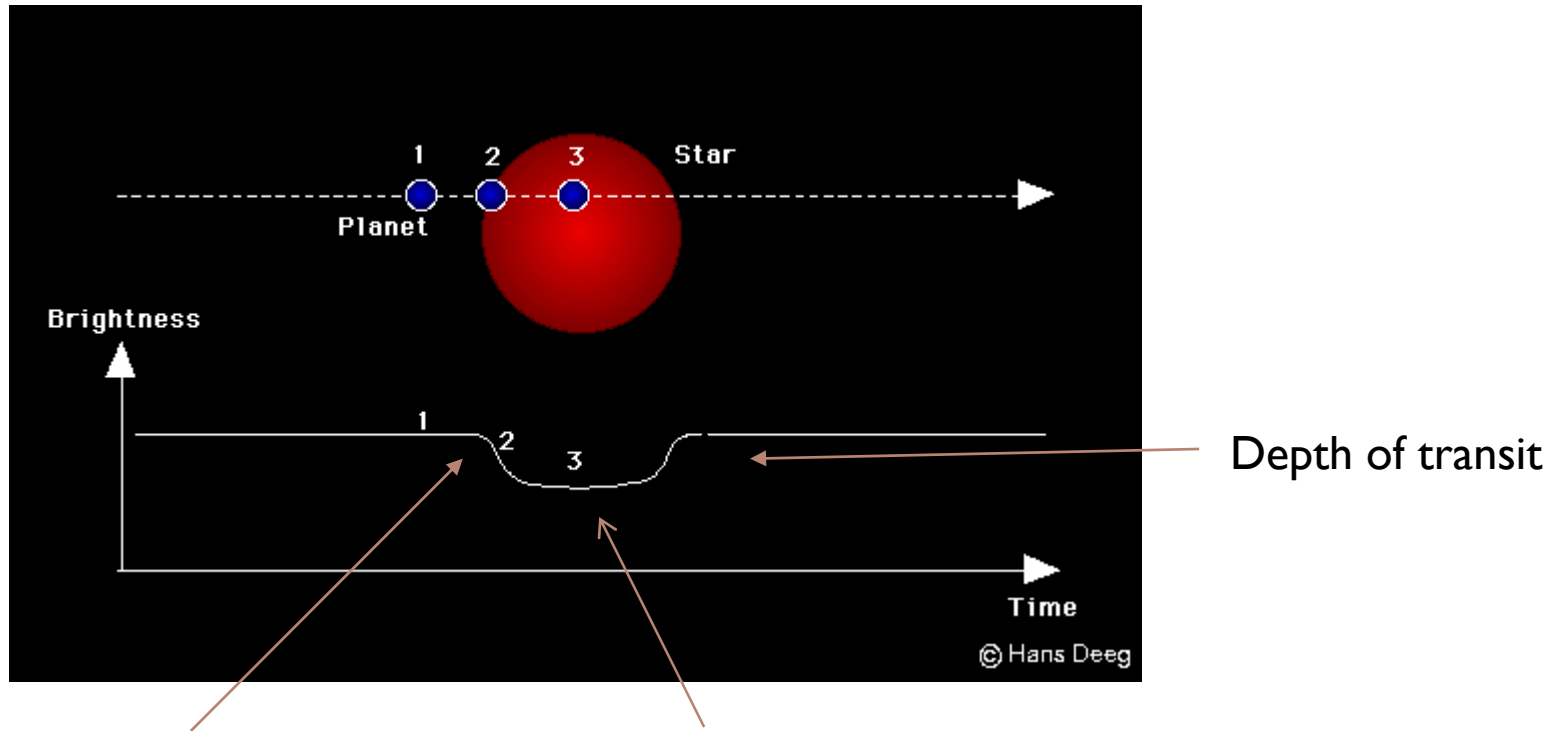
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- ▶ **Model:** a planet moving in front of a star (transit)



# Model parameters

- ▶ Parameters: four numbers:  $M=4$



Time when transit starts

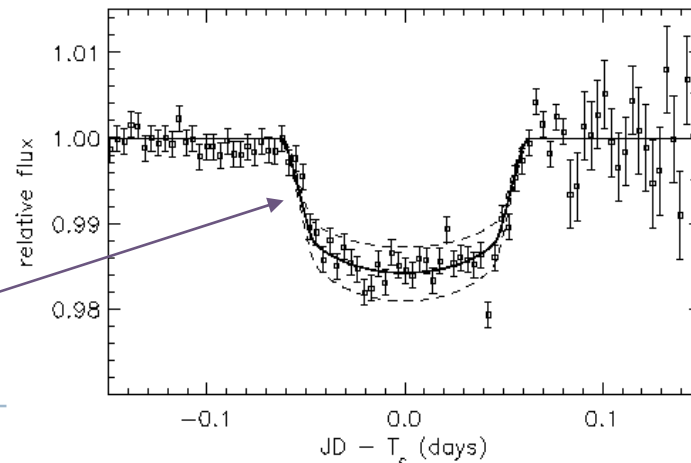
Duration of transit

Repetition period (orbit period of planet)  $T$

# Forward modelling, and noise

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- ▶ For a given set of parameters, we can predict the light curve (“forward modelling”)
- ▶ Actual data will be slightly different, because of **noise**: measurement errors, random extra light

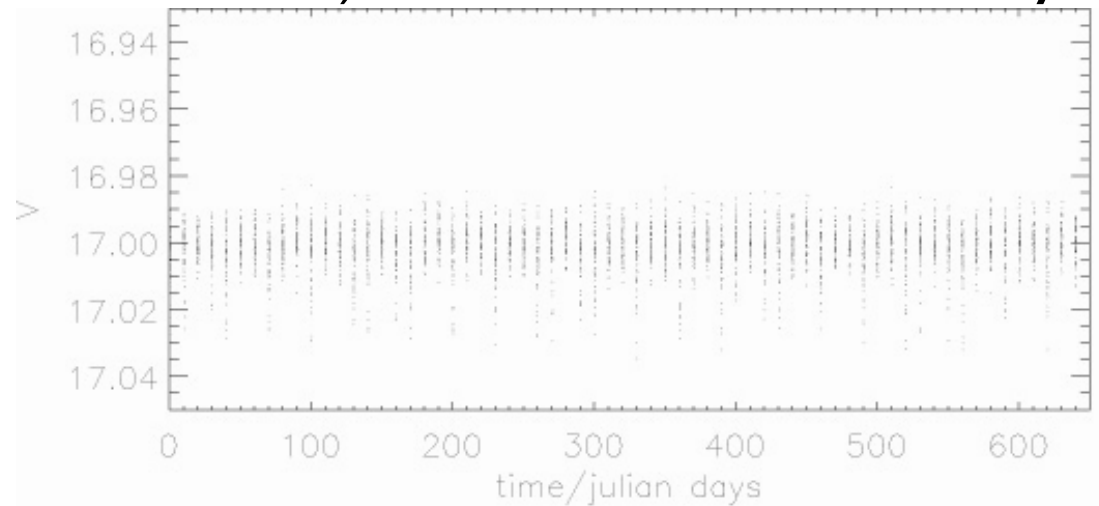


Solid line: forward model

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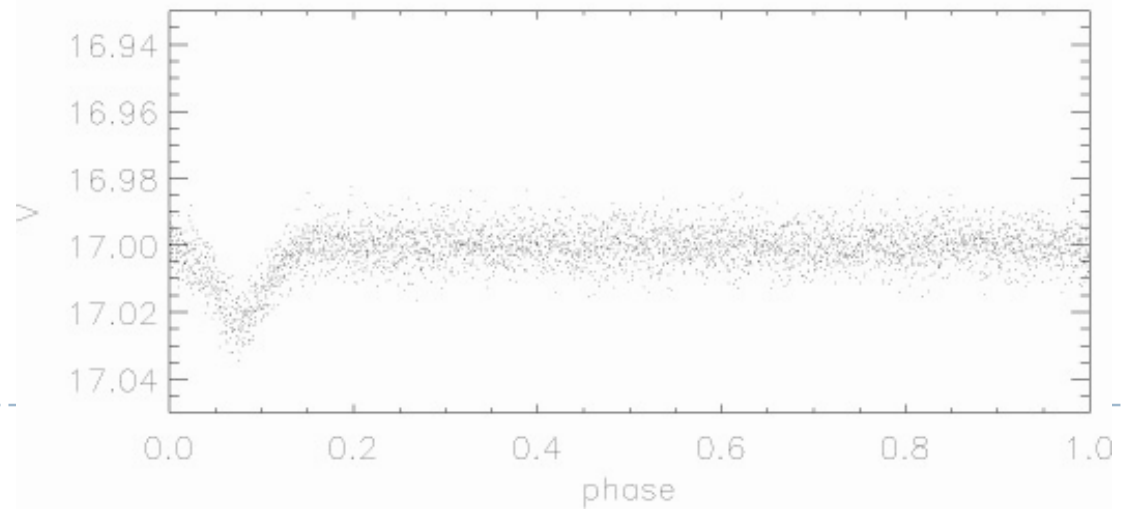
# Problem is an inverse one:

- ▶ Given the **data** (and a **model**), what are the most likely parameters?



$N=100000$

From Protopapas,  
Jimenez & Alcock  
2005



# Speed

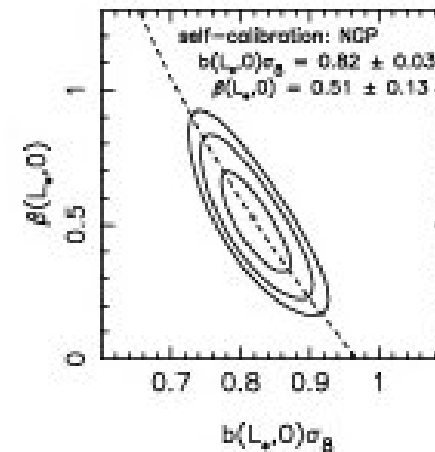
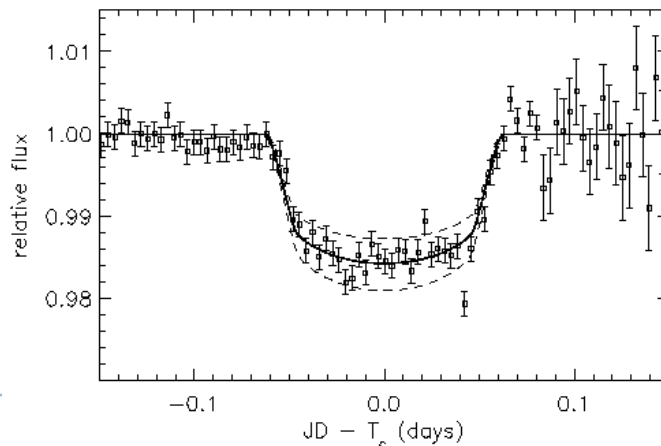
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- ▶ Likelihood calculation (for each set of parameters) requires about  $N$  operations
- ▶ If the noise is correlated, the number rises to about  $N^3$ .
- ▶ This can be very slow if there are *many items of data, or many examples to analyse*
- ▶ MOPED can speed this up to  $M$  operations
- ▶ Planets:  $N=1\,000\,000$ ,  $M=4$



# Data Compression

- ▶ Speeds up analysis by reducing the size of the dataset
- ▶ Doing this in an arbitrary way will speed things up, but will lead to a loss of accuracy
- ▶ **MOPED** is an algorithm for a massive speed up, *without* loss of accuracy
- ▶ (Even using the complete dataset, there will be errors)



# Linear compression of data

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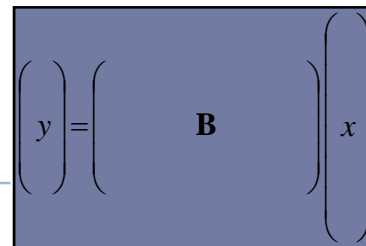
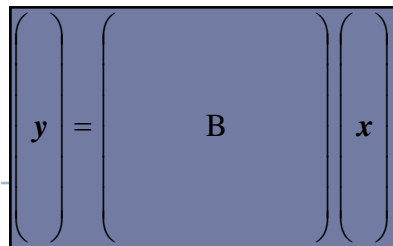
- ▶ Linear compression algorithms such as Principal Component Analysis (PCA) take

$$y = w_1 d_1 + w_2 d_2 + \dots + w_N d_N$$

- ▶  $y = \mathbf{w} \cdot \mathbf{d}$

- ▶ Many new data can be made this way, with different weight vectors  $\mathbf{w}$

- ▶ If the number of weight vectors is less than  $N$ , there are fewer  $y$  values than original data  $\mathbf{d}$ , so the data have been compressed



# MOPED properties

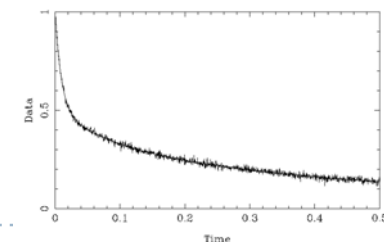
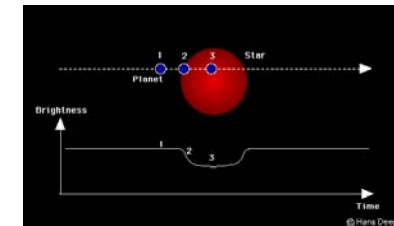
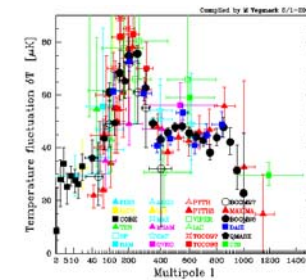
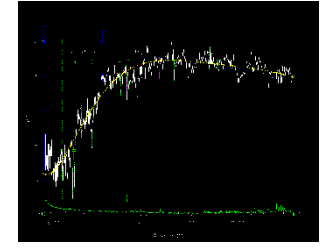
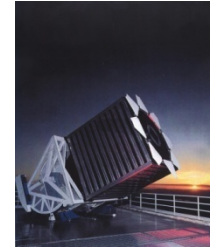
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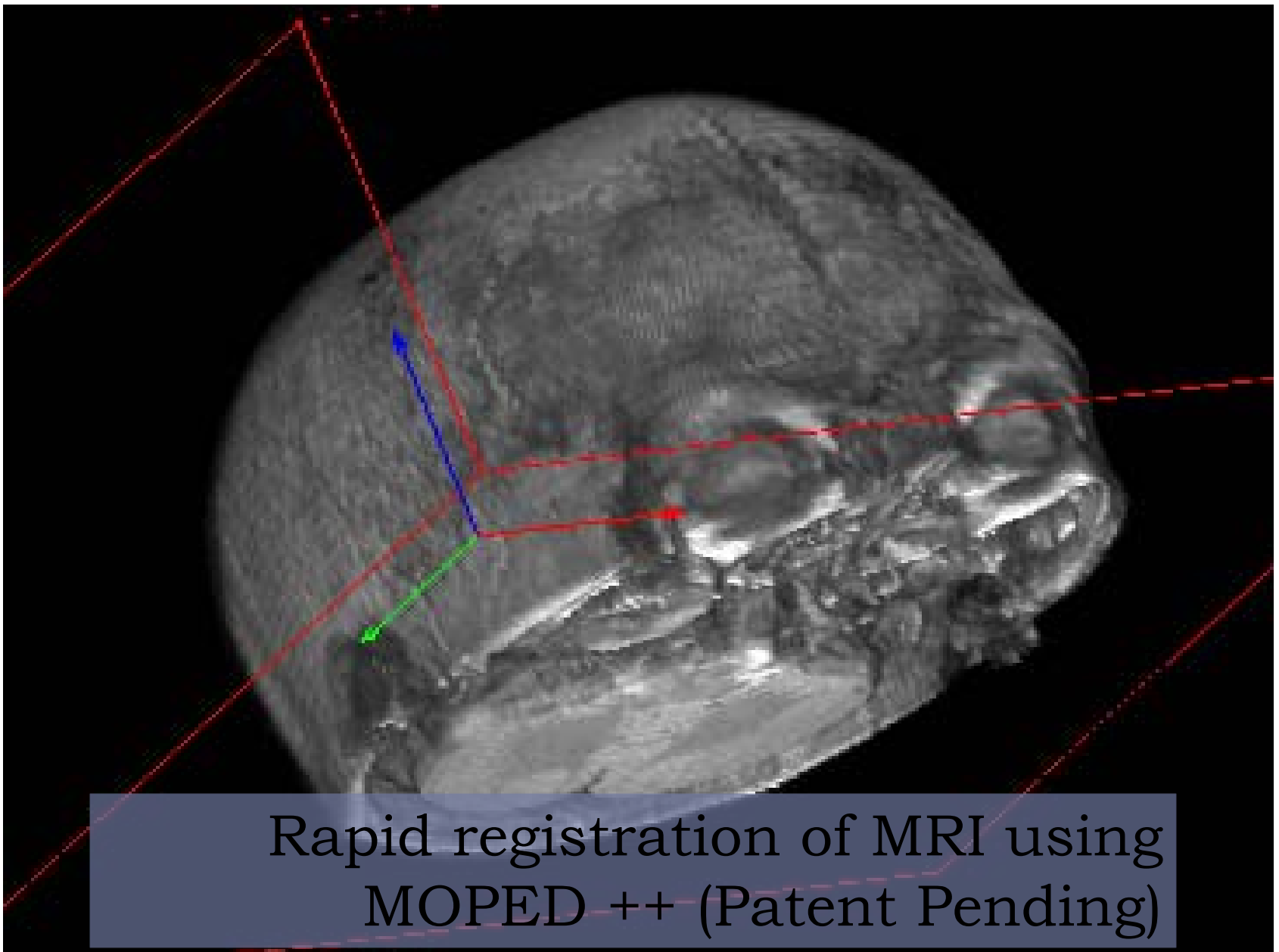
- ▶ Remarkably, the compression can be **lossless**, in the sense that...
- ▶ Model parameters are recovered **as accurately as if the entire data set had been used**.
- ▶ IT ISN'T OBVIOUS THAT THIS IS POSSIBLE!
- ▶ Data compression can be huge:
- ▶ Compression ratio = No. data/No. parameters (N/M)  
Examples: Compress to 1%, or less (cf image compression 50%).



# Examples:

- ▶ Analysis of  $\sim 10^6$  galaxy spectra
- ▶ Speed-up 100
- ▶ Microwave background fluctuation analysis
- ▶ Speed-up 100 million
- ▶ Planet transits
- ▶ Speed-up  $> 1000$
- ▶ Small-angle X-ray scattering of biological molecules



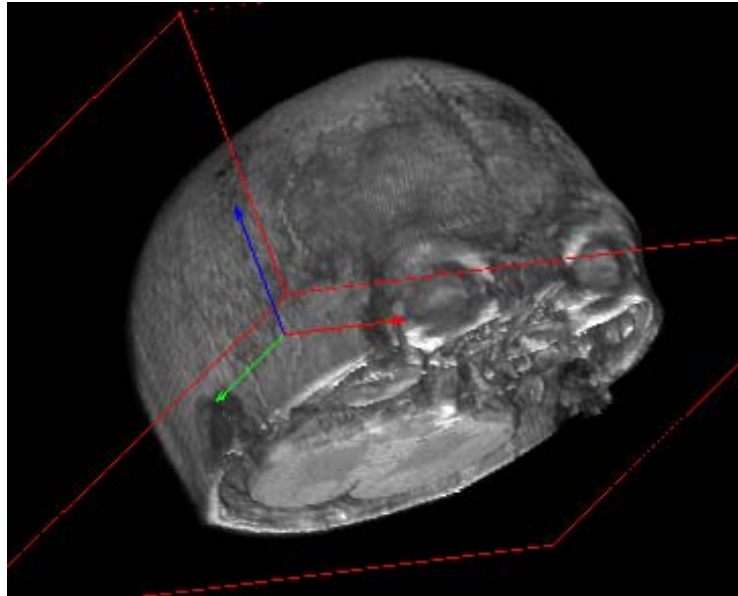


Rapid registration of MRI using  
MOPED ++ (Patent Pending)

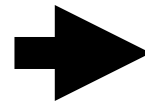
Thanks: Scottish Enterprise Proof of Concept funding.

# Difficulties with Magnetic Resonance Imaging

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- ▶ Patients move
- ▶ Scanner distorts image



Reduced scan effectiveness

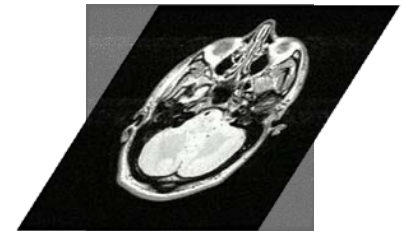
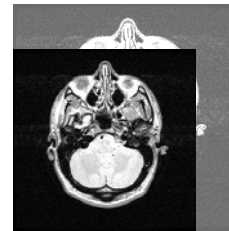
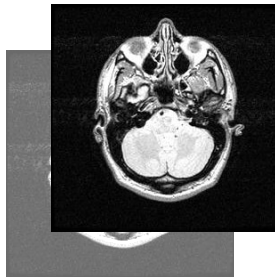
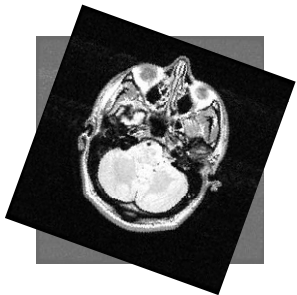
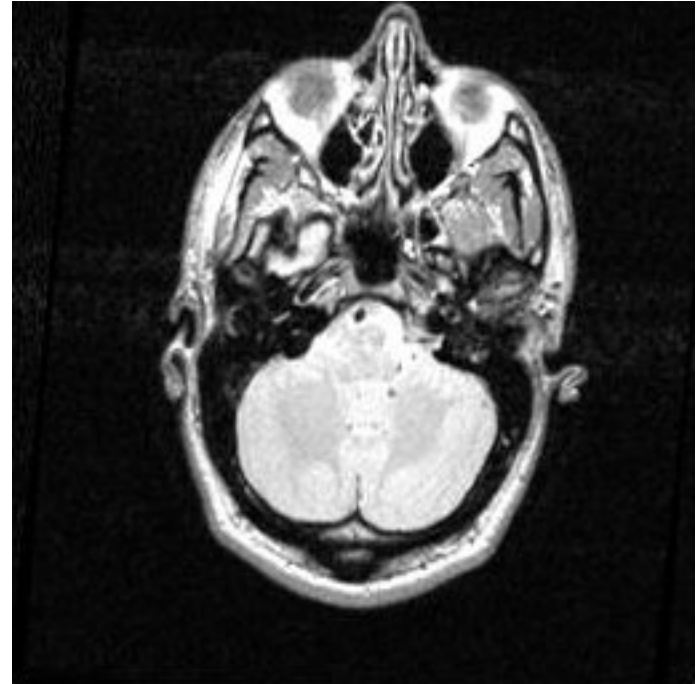
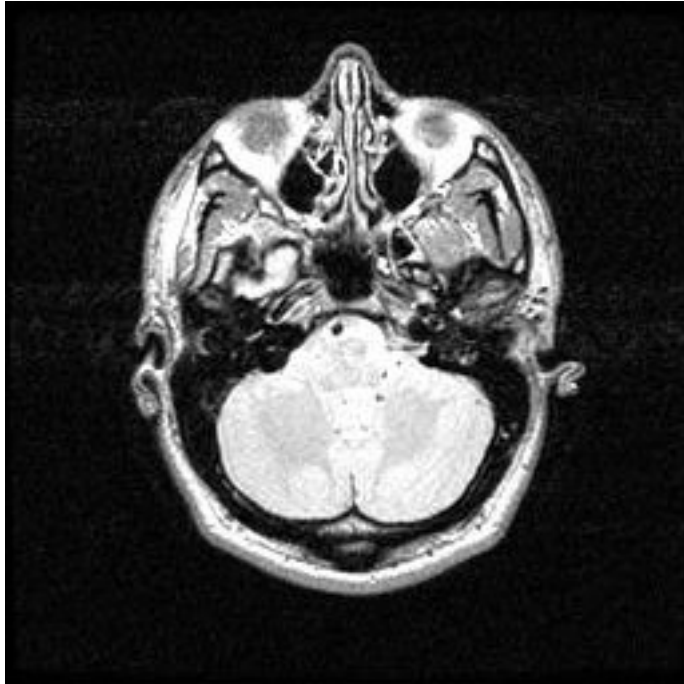
**Need to remove movement and distortions**

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# Image Distortions

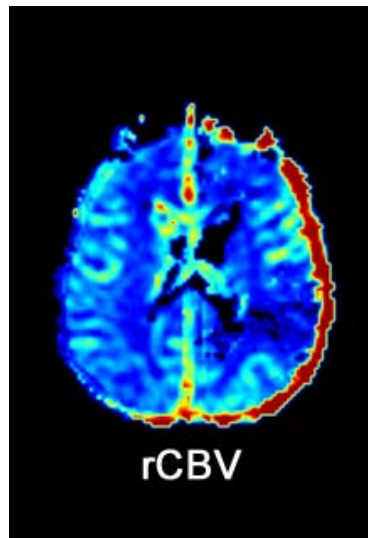
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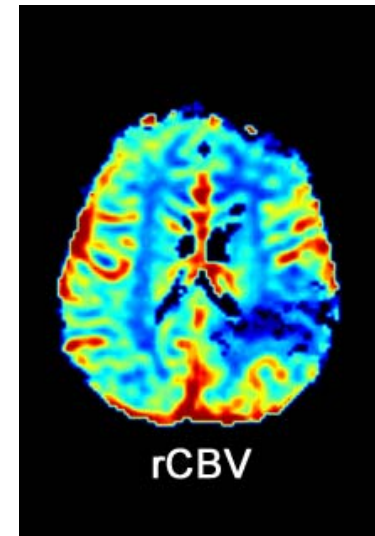
# Stroke Lesion Analysis

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Need to see fluid motion in the brain - correction required



No registration



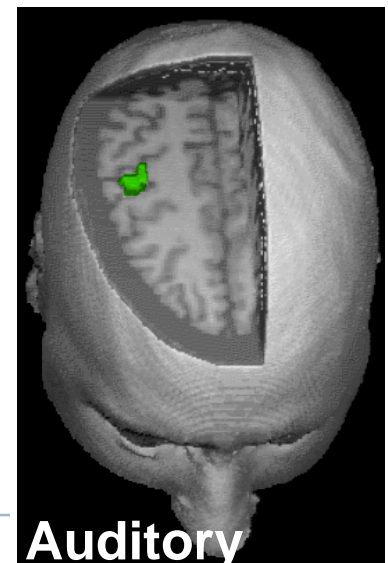
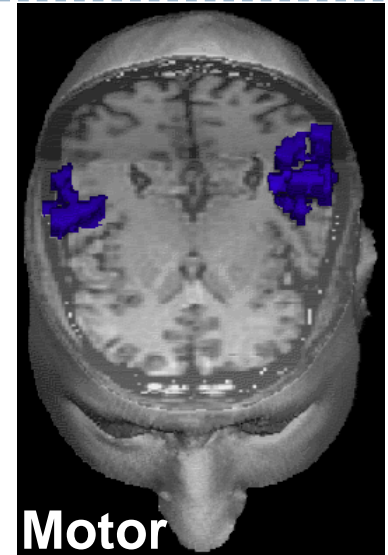
Registration



# Advanced MRI techniques

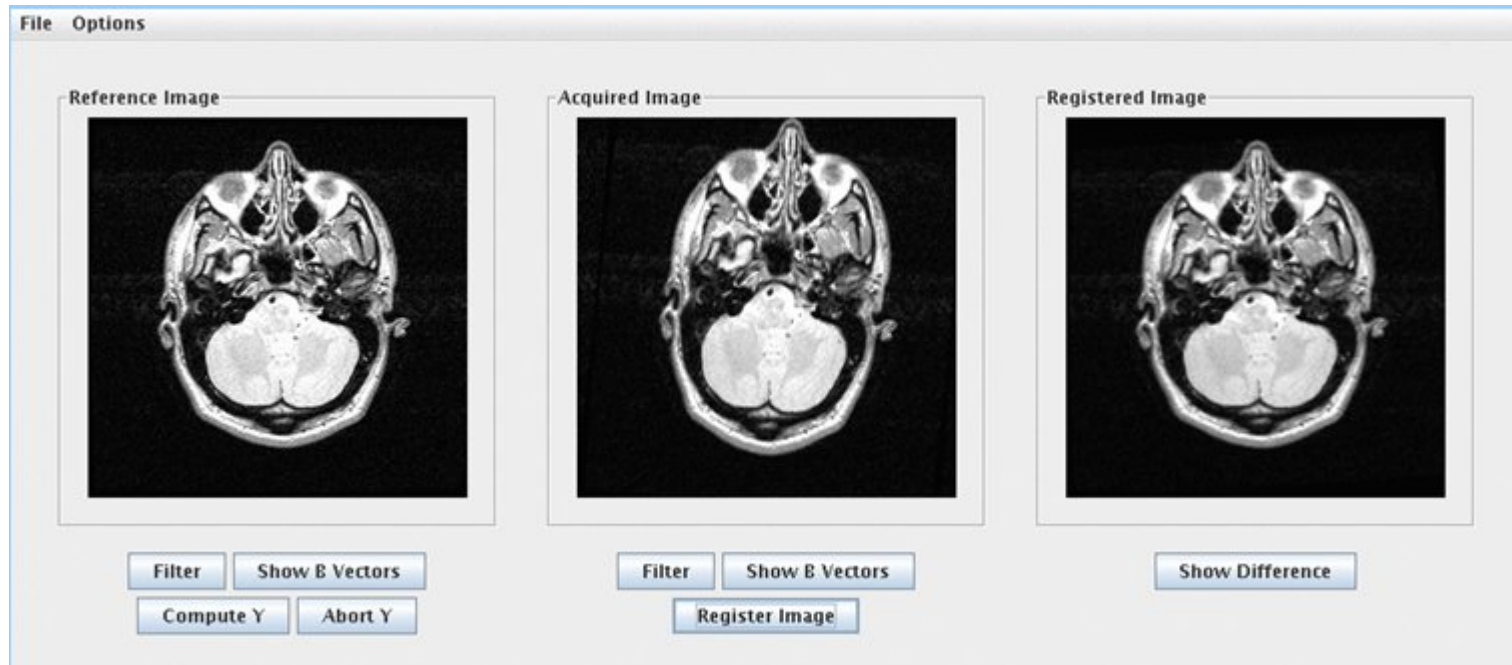
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- ▶ Allows clinical use of advanced MRI techniques (e.g. fMRI)
  - ▶ Better diagnosis
  - ▶ Find route to tumour avoiding active regions of brain (tumour pathfinding)
- ▶ Allows real-time clinical analysis
  - ▶ and scans may be repeated if necessary, without patient recall
- ▶ Removal of the distortions is a complex and time-consuming task



# 2D Registration example:

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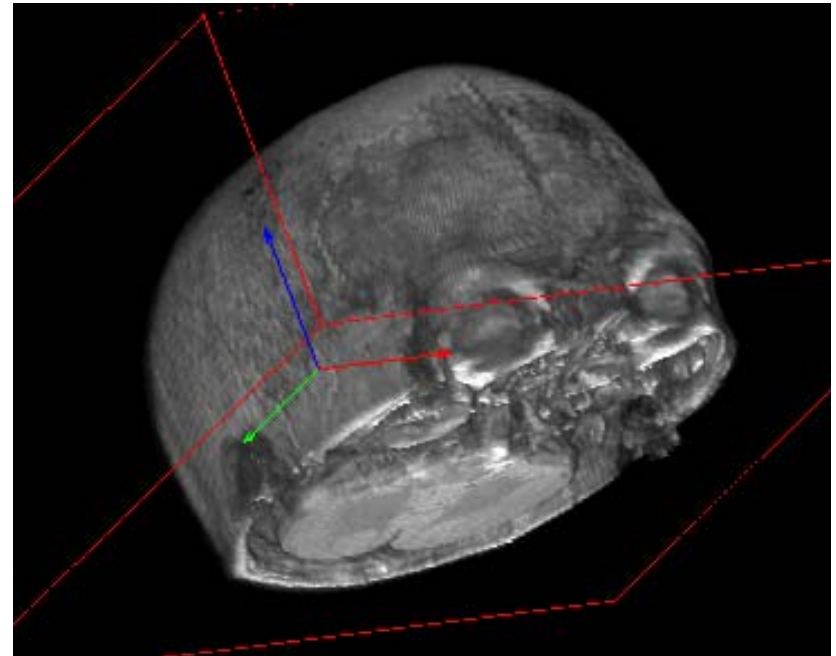
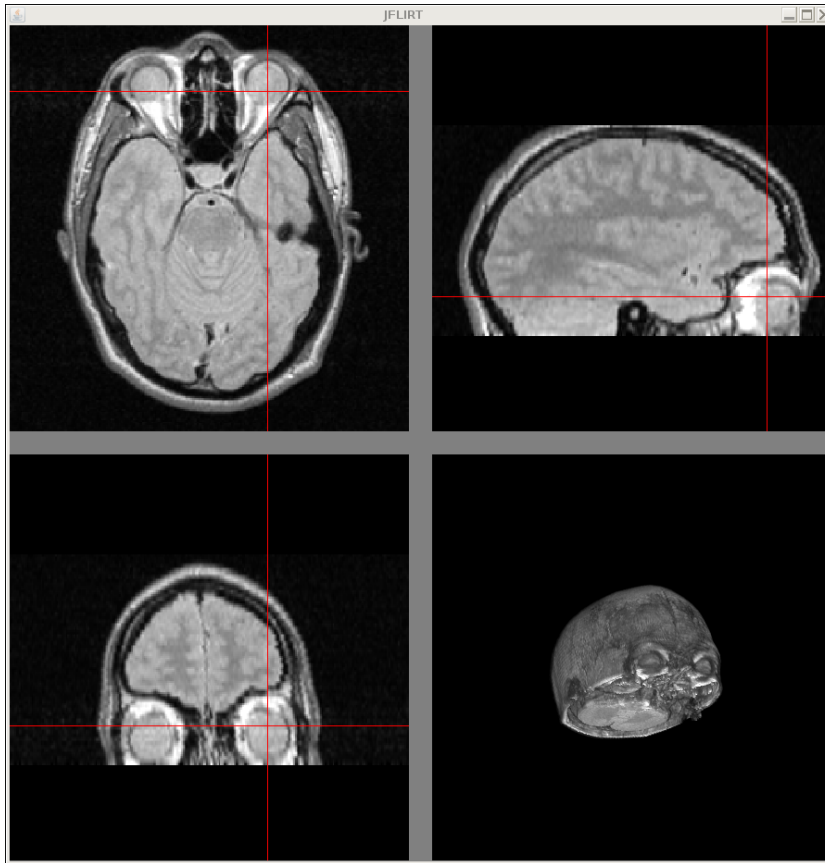
**MOPED successfully registers images in real time**

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# MOPED++ also works on 3D volumetric data

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MOPED ++ does this very fast – in real time

Full 12-dimensional affine transformation

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# Summary

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- ▶ **MOPED** can be applied to general **inverse problems**.
- ▶ MOPED can speed up analysis with *virtually no loss of accuracy* via a carefully-designed massive data compression step
- ▶ Most effective when there are many more *data* than model *parameters*
- ▶ Technology invented for rapid and accurate analysis of galaxies has been adapted successfully for Medical Imaging
- ▶ Very fast and accurate registration of MRI (and other) images, in 3D, with full 12D affine transformation, is now possible in real time
- ▶ New applications?
  - ▶ Alan Heavens. [afh@roe.ac.uk](mailto:afh@roe.ac.uk)

